Example: The Volume

<u>Integral</u>

 $\iiint g(\bar{r}) \, dv$

Let's evaluate the volume integral:

where $g(\overline{r}) = 1$ and the volume V is a sphere with radius R.

In other words, the volume V is described as:

 $\mathbf{0} \le \theta \le \pi$

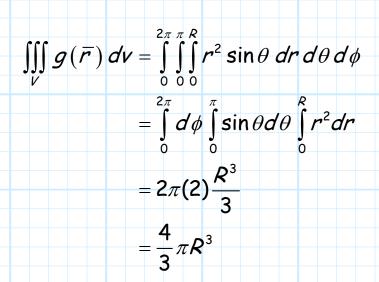
0 < r < R

 $0 \le \phi \le 2\pi$

And thus we use for the differential volume dv:

$$dv = \overline{dr} \cdot \overline{d\theta} \times \overline{d\phi} = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Therefore:



Hey look! The answer is the volume (e.g., in m³) of a sphere!

Now, this result provided the numeric volume of V only because $g(\overline{r}) = 1$. We find that the total volume of any space V can be determined this way:

Volume of
$$V = \iiint_V (1) dv$$

Typically though, we find that $g(\overline{r}) \neq 1$, and thus the volume integral does **not** provide the numeric volume of space V.

Q: So what's the volume integral even good for?

A: Generally speaking, the scalar function $g(\overline{r})$ will be a density function, with units of **things/unit volume**. Integrating $g(\overline{r})$ with the volume integral provides us the **number of things** within the space V! For example, let's say $g(\overline{r})$ describes the **density** of a big **swarm of insects**, using units of *insects/m*³ (i.e., insects are the **things**). Note that $g(\overline{r})$ must indeed a **function** of position, as the density of insects changes at different locations throughout the swarm.



Now say we want to know the total number of insects within the swarm, which occupies some space *V*. We can determine this by simply applying the volume integral!

number of insects in swarm
$$= \iiint g(\overline{r}) dv$$

where space V completely encloses the insect swarm.